Философия науки и техники
2017. Т. 22. № 2. С. 43–59
УДК: 165.0

Philosophy of Science and Technology
2017, vol. 22, no 2, pp. 43–59
DOI: 10.21146/2413-9084-2017-22-2-43-59

ИССЛЕДОВАТЕЛЬСКИЕ ПРОГРАММЫ ЭПИСТЕМОЛОГИИ

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Under What Conditions is Knowledge Critical?

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In this essay, we defend a pluralism concerning the concept of knowledge and show that the seven types of knowledge, that we distinguish, can be unified by criteria which characterize the types of knowledge as being critical. In the first part, seven types of knowledge are distinguished: 1. knowledge as true immediate objective understanding (e.g. simple logical or mathematical truths); 2. knowledge as true immediate understanding (e.g. cogito ergo sum); 3. knowledge as true objective understanding by proof (e.g. rigorous proof in logic, mathematics, and science); 4. knowledge as verification (e.g. direct and indirect observation, or experiment); 5. knowledge as justified true belief (e.g. belief in experts); 6. knowledge as possessing epistemic entropy and information (e.g. possible states that satisfy a hypothesis h, and possible states that are forbidden by h); 7. knowledge as justified corroboration (e.g. Kepler's second law is corroborated by severe tests). In the second part, conditions for knowledge to be critical are proposed. The general critical question (CQ) is: what would happen if p, a statement claimed to be known, is given up? This question, then, is applied to the seven types of knowledge. The first and second type of knowledge cannot be given up because this would lead to contradiction and absurdity. Applying CQ to the third type of knowledge may be a hint for finding an incorrect step or a gap in the proof. CQ applied to the fourth type of knowledge leads to the critical investigation, whether the verification is a reproducible effect, whether it is observer-invariant, and whether it has a suitable interpretation with a well-corroborated hypothesis or law. Applying CQ to the fifth type of knowledge forces to check the reliability of the experts or one's own reliance. Moreover, it leads to uncover the hidden assumptions and preconditions. Applying CQ to the sixth type of knowledge guides us to investigate the excluded possible states and, consequently, to consider alternatives. For example, Euclidean Geometry excludes positive or negative curvature; an alternative (hyperbolic) geometry with negative curvature was discovered by Lobachevsky and Bolyai. When applying CQ to the seventh type of knowledge, we have to distinguish two cases. Very well confirmed laws of nature or the fundamental constants of nature behave similar to the first type of knowledge: Absurd consequences follow, if they would be given up. Alternatively, applying CQ to corroborated scientific hypotheses often leads to revision, sometimes to refutation.

Keywords: types of knowledge, epistemology, conditions for knowledge

1. Seven Types of Knowledge

There is the restricted view that knowledge only exists in the form of well justified true belief, often connected with a state of mind that we are absolutely sure of something. The concept of knowledge as well justified true belief is widely accepted in the philosophical history, beginning with Plato's Theaetetos until the middle of the 20th century. Gettier became well-known for his counterexamples showing that knowledge is not always justified true belief [cf. Gettier 1963]. Although widely accepted, credited and discussed, Gettier's examples are not very convincing, as they are very artificial and either based on irrelevant inferential moves, or not a real counterexample at all[1]. On the other hand, there are many realistic counterexamples in the history of science showing that knowledge is not always justified true belief; e.g. Fermat's conjecture that there are no solutions for $x^n + y^n = z^n$ where $n > 2$, which was justified true belief, but not considered knowledge until Wiles finished his proof in 1994. Or the prediction of the deviation of light rays caused by huge masses by the Theory of General Relativity of 1916 by Einstein, but not knowledge before it was confirmed by the expeditions of the Royal Society to South Africa and Brazil observing the effect during an eclipse in 1919. It was justified true belief.

We will see that justified true belief is only one type of knowledge, for there are other types which will be discussed in the first section of this paper. In the second section, we discuss conditions under which each type of knowledge is critical.

1.1. Knowledge 1: True immediate objective understanding

We understand under true immediate objective[2] knowledge very simple transparent truths in the sense of axioms, i.e. principles "which are true, primitive, immediate, more familiar than and prior to and explanatory of the conclusion... and such which are necessary to grasp for anyone who is going to learn anything", as Aristotle says [Aristotle 1985 (APost), 71b21][3]. Examples are principles of propositional calculus like the principle of non-contradiction $\neg (A \wedge \neg A)$, or $A \rightarrow A$ and $A \vee \neg A$ as in "If the sun is shining then the sun is shining" or "The world is finite or the world is infinite"; or simple mathematical truths like $2 + 2 = 4$, $2 \cdot 3 = 6$ and $7 - 2 = 5$ of the multiplication table or general ones like $x = x$, or $x = y \rightarrow s(x) = s(y)$, where $s(x)$ is the successor of x. Leibniz calls such axioms "primitive truths of reason" and characterizes them as follows: "The primitive truths of reason are those which I call by the general name of identicals... Those which are affirmative are such as the following: everything is what it is, and in as many examples as we may desire, *A* is *A*, *B* is *B*" [Leibniz 1978 (NE), p. 4, 7, 10]. The definition of knowledge 1 (K_1) is:

Def. 1: A person a knows (K_1) that p (for short: $a\ K_1\ p$) iff
p is a simple logical principle, or
p is a simple mathematical truth.

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1 For a critical discussion of Gettier's examples see [Weingartner 1996, p. 227 ff].

2 Objective knowledge here means that it is independent of individual circumstances, but not independent of human reason, as Frege suggested [cf. Frege 1884/1977, p. 72].

3 If we drop the last part "and such... learn anything" Aristotle calls it a postulate, with this part an axiom.

1.2. Knowledge 2: True immediate intrasubjective understanding

As Descartes points out correctly, there are also some primitive and immediate empirical truths: "Thus each individual can perceive by intellectual intuition that he exists, that he thinks..." [Descartes 2009, p. 3].

Knowledge 2 is immediate as are all judgments of introspection such as "I am now thinking about my essay." It is intrasubjective insofar as the empirical proposition can be understood by everybody in nearly the same way, although every person understands it for him-/herself. Famous examples of such primitive and immediate empirical propositions are the "fallor ergo sum" of Augustine or Descarte's "cogito ergo sum". The second type of knowledge can be defined as follows:

Def. 2: $a\ K_2\ p$ iff
p is a simple empirical statement about one's own existence, or
p is a simple statement of introspection.

1.3. Knowledge 3: True objective understanding by proof

"To prove" can be understood in two ways: First, as to prove in the strong sense of a logical or mathematical proof. And second in the sense of verifying empirical individual facts (cf. 1.4 below).

Here, *understanding by proof* is meant in the strict sense of mathematical or logical proofs. The person who proves a certain logical or mathematical statement *knows* that this statement follows logically from certain premises. And he *knows* further by the general principle of logical consequence: if these premises are true, then the conclusions are true also, since all logical consequences of true statements are true. The principle of logical consequence was precisely formulated first by Bolzano [Bolzano 1914 II, § 155] and then by Tarski [Tarski 1936; 1956, p. 409-420]. For proofs in mathematics logical rules of deduction, like *modus ponens*, *hypothetical syllogism* and others – which all satisfy the principle of logical consequence – are not sufficient. Further principles like that of mathematical induction or presuppositions like the axiom of choice, and sometimes very sophisticated ones like the FAN-principle or the forcing-method have to be applied. This type of knowledge can be extended to other persons who either found another way to prove the mathematical statement in question or have controlled the proof step by step. The possibility to control such proofs is an important condition to call this knowledge *objective*. Famous recent examples of such knowledge is Wiles' proof of Fermat's last theorem (1994) or Matiyasevich's answer to the 10th Problem of Hilbert (1970), or Perelman's proof of the Poincaré Conjecture (2004).

The difference between this type of knowledge and knowledge of facts is well expressed by Wittgenstein: "A proof ought to show not merely that this is how it is, but this is how it has to be" [Wittgenstein 1956 II, § 1, 9]. Knowledge 3 can be defined thus:

Def. 3: $a\ K_3\ p$ iff
a is able to carry out a logical proof of p, or
a is able to carry out a mathematical proof of p.

1.4. Knowledge 4: Verification

This type of knowledge is similar to K_3, in the sense that it is another type of proving. But it is different in that it is concerned with empirical facts. What is meant is an "empirical proof" by direct observation, indirect observation or scientific experiment. These modes of verification are available to different domains of observers: While direct observation, like witnessing a car accident, is available for all human adults under normal conditions, indirect observation, like examining something under the microscope or telescope, and scientific experiments are only available for specially trained and educated people or experts in the field of science. Several assumptions are involved here: a certain kind of observer-invariance relevant for the respective situation, a reproducible effect in case of indirect observation and scientific experiment, a suitable interpretation of the observational data with the help of scientific hypothesis or theory (cf. 2.2.4 below).

Knowledge 4 can be defined as follows:

Def. 4: $a\ K_4\ p$ iff
(1) a is able to verify p by direct observation, or
(2) a is able to verify p by indirect observation, or
(3) a is able to verify p by some scientific experiment.

1.5. Knowledge 5: Justified true belief

This type is, as mentioned, a very frequently occurring type of knowledge. With the exception of K_1 and K_2, most of the other types of knowledge can be connected with K_5. Especially K_3 (true objective understanding by proof), K_4 (verification) and K_7 (justified corroboration) come to mind in connection to knowledge as justified true belief because we often see the conclusions of proofs or the verification and corroboration processes as justifications for believing that these obtained conclusions are true. However in a more ordinary view, justified true belief seems to occur mostly when we learn something from others. We regard other people as trustworthy and believe that their knowledge is reliable, because in most cases, we cannot prove it ourselves. But even if we could prove it ourselves in principle, it is often necessary to believe others because time is too short. This justified true belief begins with our birth: children believe their parents and teachers. Later it occurs in a more sophisticated way when scientists believe their colleagues.

However, there are cases where knowledge is not justified true belief.

First of all, we think that knowledge in the sense of K_1 and K_2 are not types of belief but types of understanding, although a belief may follow from the understanding or grasping. These are cases of knowledge which are not justified true belief. Secondly there are cases of justified true belief which are not (or not yet) cases of knowledge: Real counterexamples are famous scientific conjectures and predictions which were proved later and could be called knowledge only after they have been proved (cf. section 1 above). So although knowledge as justified true belief has a wide field of application, it cannot be used as a general definition of knowledge. We define this type of knowledge as follows:

Def. 5: $a\ K_5\ p$ iff

(1) a believes that experts b, c, d, ... have K_3 or K_4 or K_6 or K_7 concerning p and therefore a believes that p, or
(2) a has himself K_3 or K_4 or K_7 concerning p, or
(3) a is an expert concerning p and has K_3 or K_4 or K_6 or K_7 concerning p and therefore believes that p.

1.6. Knowledge 6: Possessing epistemic entropy and information

Let us begin with two examples:
A... Keith will arrive in Graz within December 1 and 6, 2017.
B... Keith will arrive in Graz on December 3, 2017.
We now consider the number of possible real states that satisfy the above statements. We can easily grasp that the number of possible real states that satisfy statement A is higher than that of statement B. We call the number of possible real states that satisfy a statement p the *epistemic entropy* of p (for short: $EE(p)$).

Def. 6.1: $EE(p)$ = the number of possible real states that satisfy p.
Possible real states are understood as having the following properties:
(a) They are compatible with well confirmed laws and constants of nature.
(b) They are finite, because we assume that the universe is finite too (according to the General Theory of Relativity).
(c) Possible real states are contingent and last a short time interval such that a measurement or observation is possible. Thus, a possible real state is a short event.
(d) The statements representing them are restricted by relevance conditions, which do not permit the usual closure conditions: Negation and disjunctions are de dicto and not de re. There are neither negative possible real states nor disjunctive ones.
(e) Possible real states can be described or represented by *basic statements* in the sense of Popper [Popper 1969, § 27–29] satisfying condition (a). The general form is: state s occurs at space time interval $\Delta (x_1\, t_1 \mid x_2\, t_2)$.
(f) Basic statements are not invariant w. r. t. transformations of logical equivalence and do not obey the usual logical closure conditions. For example if p is a basic statement then $(p \wedge q) \vee (p \wedge \neg q)$ is logically equivalent to p but is not a basic statement. Similarly not every logical consequence of a basic statement is again a basic statement. Thus neither $p \vee q$ nor $\neg p \rightarrow q$ are basic statements although they follow logically from p.

If we consider statements about single events like A and B above, then increasing information goes together with decreasing epistemic entropy, i.e. B contains more information than A. For such cases we could therefore define epistemic information as 1/epistemic entropy. However, we want to also include scientific laws which are represented by universal statements. And scientific laws are satisfied by a huge number of possible real states. Consequently, such a definition is not suitable. We propose therefore that the epistemic information of p will be defined by the number of *possible states* that are excluded by p. Since we required for *possible real states* that they are compatible with laws of nature and fundamental constants of nature – condition (a) above – we have to drop this restriction of compatibility for those states which are excluded and call them *possible states*. They satisfy the following conditions:

(1) Possible states are compatible with laws of First Order Predicate Logic with Identity and with the laws of mathematics.
(2) They are not restricted to be finitely many.
(3) Possible states are singular, not compound and can be represented by atomic statements.
(4) Possible states are contingent.
(5) The statements representing them satisfy conditions (d) and (f) of properties representing possible real states.

The epistemic information of p (for short: $EI(p)$) is the number of possible states that are excluded by p.

Def. 6.2: $EI(p)$ = the number of possible states that are excluded by p.

Looking at our statements A and B which describe singular events, we can now grasp that:

$EE(\mathrm{A}) > EE(\mathrm{B})$,
$EI(\mathrm{A}) < EI(\mathrm{B})$,
$EI(\mathrm{B}) > EE(\mathrm{B})$.

On the other hand, we can look on scientific laws. As an example we take Kepler's Laws:

L_1: All planets move in ellipses.
L_2: In equal times the radius-vector passes over equal surfaces.
L_3: T^2/a^3 = constant.

We understand immediately that such laws have both huge epistemic entropy and large epistemic information. By applying the above principle we get that

$EE(L_1\ L_2) > EE(L_1\ L_2\ L_3)$,
$EE(L_1\ L_2\ L_3) > EE(L_1\ L_2\ L_3\ m,\ d)$, where m, d are the boundary conditions of masses and distances;
$EI(L_1\ L_2\ L_3\ m,\ d) > EI(L_1\ L_2\ L_3)$.

Adding the boundary conditions of the masses of the planets and their distances from the sun decreases the epistemic entropy and increases the epistemic information.

We define knowledge K_6 as follows[4]:

Def. 6: $a\ K_6\ p$ iff
(1) a possesses $EE(p)$ and
(2) a possesses $EI(p)$.

1.7. Knowledge 7: Justified corroboration

Since universal hypotheses, laws or theories cannot be verified, Popper found a way out by suggesting corroboration for hypotheses, laws or theories in the following sense: they have to withstand severe and crucial tests and should make new predictions which could be tested by indirect observation or experiment [cf. Popper 1959, Appendix *IX, and Popper 1963, pp. 220, 242]. Popper used *corroboration* instead of *confirmation* since the latter might suggest that the confirmation can be final. There is a huge amount

[4] This type of knowledge was originally proposed in [Weingartner 2014, p. 286ff]. A detailed exposition is contained in [Weingartner 2017].

of literature about the *problem of confirmation* of scientific hypotheses, laws and theories. We do not go into detail since the aim here is only a short characterization in order to be used concerning the conditions for types of knowledge to be critical.

Further we might say that one theory A is better corroborated than another B if both are about the same domain and A has withstood more severe and crucial tests than B. Thus, Newton's theory of planetary motion is better corroborated than Kepler's (especially beyond the first four planets, since concerning these the measurement results are almost the same).

Moreover, we can say that one theory A is nearer to the truth (or a better approximation to the truth) than another B if A has more true consequences and less false consequences than B [Popper 1963, Appendix, and Popper 1972, p. 330ff][5].

To sum up, corroboration contains the following parts:

Either hypotheses or some laws (the latter may be the gist of some theory), consequences (may be as predictions) which are deduced from the hypotheses or laws together with initial and boundary conditions, and severe and crucial tests of these consequences.

If the tests are positive the hypotheses or laws are corroborated.

However, an important difference or inhomogeneity should not be overlooked concerning the domain of knowledge as justified corroboration. In some cases the corroboration or confirmation is rather strong if not final in a good sense: take the laws of conservation of energy (in a closed system) or the law of entropy. Einstein thought that the law of entropy will never be overthrown:

"Therefore the deep impression which classical thermodynamics made upon me. It is the only physical theory of universal content concerning which I am convinced that, within the framework of the applicability of its basic concepts, it will never be overthrown... On the basis of the kinetic theory of gases Boltzmann had discovered that, aside from a constant factor, entropy is equivalent to the logarithm of the 'probability' of the state under consideration... This idea appears to be of outstanding importance also because of the fact that its usefulness is not limited to microscopic description on the basis of mechanics." [Einstein 1949, p. 33, 43].

Or take the value of the fundamental constants of nature like c, G, h, m_p/m_e, α (constancy of light velocity in vacuum, gravitational constant, Planck's constant, ratio of proton-mass to electron-mass, fine-structure-constant).

We may call this knowledge 7.1 ($K_{7.1}$).

On the other hand, there are cases where the corroboration is sufficiently strong to work with the respective hypothesis or law. An example would be the Hardy-Weinberg law about genetic frequencies. It cannot have a very strong corroboration because it assumes five antecedence conditions which can hardly be satisfied (no genetic drift, random mating populations, no unbalanced mutation or migration, no selection). We may call this knowledge 7.2 ($K_{7.2}$).

An example for a simple corroboration is again taken from Kepler: the consequences from his second law together with initial and boundary conditions predict that a planet moves with higher velocity when it is closer to the sun rather when it is farer away from it (representing K_3), which can be corroborated by test-observations with concrete planets (representing K_4).

5 In order to avoid the objections of Tichy and Miller (against this definition) one has to replace "consequences" by "relevant consequence dements" as has been shown in [Schurz/Weingartner 1987].

Knowledge 7 is therefore defined as follows:

Def. 7: $a\ K_7\ p$ iff
p is a scientific hypothesis or law-statement, and
a believes that there are experts b, c, d, ... who
possess K_3 of consequences from both p and initial and boundary conditions, and
possess K_4 (or K_2, K_3) of these consequences, and therefore believes p to be true or approximately true.
a him-/herself possesses K_3 and K_4 in the above sense.

The above formulation is one for dynamical laws. A similar formulation can be given for statistical laws[6].

2. Conditions for knowledge to be critical

We shall first propose a very general critical question and then show that, when applied to the seven types of knowledge, it differentiates immediately two groups: K1 and K2 on the one hand from K3 to K7 on the other (2.1). In a second part (2.2) we shall show how, i.e. with which consequences, the critical question applies more accurately to the seven different types of knowledge.

2.1. The general critical question

If p is the statement known, the general critical question is:

CQ What would happen if p is given up?

When applying this question to the seven types of knowledge, we proceed like in an "Indirect Proof". The method of indirect proof is a well-known method in mathematics: if one wants to prove the statement A, one assumes – just for the sake of proof – non-A. Then one tries to derive absurd consequences, a contradiction, from non-A. If this is possible, then A has been proven. If not, the reason may be either that the derivation of a contradiction was unsuccessful (one was, so far, not able to derive it) or no indirect proof is possible because the respective statement is contingent.

We see immediately that applying *CQ* with the method of indirect proof to the seven types of knowledge will lead to splitting up *CQ* into two sub questions depending on whether we apply *CQ* to K_1 and K_2 or to K_3 - K_7.

Applying *CQ* to K_1 and K_2:

If we apply *CQ* to K_1 then the assumption of the indirect proof is the negation of a simple principle of logic like $p \rightarrow p$ (if the sun shines then the sun shines) or of a simple principle of mathematics like that for any two natural numbers n and m it holds: $m = n$ or $n < m$ or $n > m$. The absurd consequence searched for frequently by mathematicians using the indirect proof is $0 = 1$.

From such an assumption one can easily derive a contradiction (absurdity). This shows that one cannot give up these principles. Therefore, a respective sub question concerning such a principle p might be:

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6 For conditions of dynamical and statistical laws cf. [Mittelstaedt/Weingartner 2005, p. 150, 154].

CQ_1 What should happen if p cannot be given up?

And the answer to this subsequent is an answer to scepticism: one can show that every denial or doubt about such simple principles leads to some or other absurdities. A similar result obtains if we apply CQ to K_2. The respective assumption of the indirect proof denies the own existence or self-consciousness and introspection. Such absurdities, as the denial of one's own existence, are also fatal to extreme scepticism.

Applying CQ to K_3 - K_7:

If we apply CQ to K_3 - K_7, then the assumption of the indirect proof need not to lead to an absurdity. In case of K_3 one may find somewhere an incorrect step or gap in the proof. In such cases the denial that the conclusion follows from the premises is correct, but does not lead to absurdity. However, if the proof is valid the assumption of the indirect proof will lead also here to contradiction or logical absurdity. When CQ is applied to K_4 - K_7 then the assumption of the indirect proof does not lead to contradiction or logical absurdity, but it may lead to an empirical absurdity or incompatibility with well-established empirical facts (recall section 1.7).

Therefore a respective sub question concerning a claim q of validity (K_3) or truth or approximate truth (K_4–K_7) might be:

CQ_2 q can (at least in the sense of logical possibility) be given up. Assume q is given up. What are the consequences?

The answer to this sub question is different for the five types of knowledge K_3–K_7 and will be given in the next section.

2.2. Special critical conditions for the different types of knowledge

We want to point out from the beginning that the widely accepted necessary conditions for knowledge namely "what is known is true" (KT) will not be presupposed for all seven types of knowledge. This condition is certainly satisfied for K_1 and K_2. It will be also satisfied by K_3 and K_4 if a critical examination has been applied for them. However, it cannot be generally presupposed for K_5–K_7. Why speak of knowledge in these cases then, we might ask. The reason we propose is this: Most of what we "know" through contemporary science is based on K_4–K_7. For K_4, condition KT is satisfied after critical examination. As has been mentioned in section 1.5 above, most of what scientists know, they know from colleagues and other scientists, i.e. as justified true belief which is in most cases reliable, that is, true (K_5). All empirical scientific knowledge is possessing epistemic entropy and epistemic information. This type of knowledge (K_6) is especially important for scientist's understanding of laws. Finally, corroboration of hypotheses, laws and theories (K_7) is essential for the knowledge of scientists.

Therefore, to deny knowledge for K_5–K_7 would mean to throw out the most important domains of knowledge belonging to contemporary science and to keep only the most simple and trivial parts (K_1 and K_2) – which certainly have to be presupposed and must not be forgotten. Also K_3 is necessarily presupposed for scientific proofs and K_4 is necessary to get data. But contemporary science cannot be built just on K_1–K_4. This basis is much too small. Since hypotheses, laws and theories go far beyond K_1–K_4 they cannot be reduced to that[7]. This is the reason why we do not want to throw out that most important part of science from the domain of scientific knowledge.

We are now applying the critical questions to the seven types of knowledge.

2.2.1. K_1: True immediate objective understanding

As an example we take the principle of non-contradiction (PNC) in a most tolerant form: a sentence and its negation cannot both be true, or, at most one member of the pair of statements p, $\neg p$ can be true. Observe that this formulation which appears also in Aristotle's metaphysics [Aristotle 1985 (Met), 1011b 13 and 1062a 22] allows many-valued logics.

We apply now *CQ* and CQ_1 to PNC in order to see what bad consequences result from a denial of PNC. Such consequences were discussed already by Aristotle in book IV (gamma) sections 4–6.

"There are some who [...] declare it possible for the same thing both to be and not to be [...]. However, the proper beginning for all such debates is not a demand for some undeniable assertion [...] but a demand for expressing the same idea to oneself and to another [...]. If a person does not do even this, he is not talking either to himself or to another. If he grants this there can be demonstration; for then something definite will be proposed. And he who concedes this, has conceded that something is true even without demonstration. Thus, not everything can be so and not so." [Aristotle 1985 (Met), 1006a 1–29].

That means that someone who denies PNC may either say nothing, then there is no conversation possible; or he/she may say something *p* and mean also non-*p*, but then he/she cannot be taken seriously and there is no conversation possible; or eventually the person may say something definite (i.e. not at the same time its negation) then he/she has conceded that something is true and not false, which means accepting PNC.

2.2.2. K_2: True immediate intrasubjective understanding

If we apply *CQ* to the *fallor ergo sum* or to the *cogito ergo sum* this leads to what Jaakko Hintikka called an "existential inconsistency":

We shall say that *p* is *existentially inconsistent* for a person referred to by *a* to utter if and only if the longer sentence '*p* and *a* exists' is inconsistent in the ordinary sense of the word [Hintikka 1962, p. 11][8].

Applied to the fallor ergo sum this means: the sentence "I am mistaken but I don't exist" is existentially inconsistent since the longer sentence "I am mistaken but I don't exist and I exist" is inconsistent.

7 Cf. the lucid arguments in support of that in [Popper 1963, p. 186ff].

8 For a discussion of Hintikka's proposal cf. [Weingartner 1966, p. 302ff].

2.2.3. K_3: True objective understanding by proof

Knowledge by proof means first of all that the proof is valid, i.e. that the conclusion follows from the premises. And it may mean in addition that the conclusion is true if the premises are self-evident or have been proven already from unquestionable other premises.

Applying *CQ* therefore means asking what happens if the proof is invalid. This can be so in two cases: First, if the respective conclusion is unprovable (either in general or just from these premises), or second, if the inference contains some invalid step or some gap although the respective conclusion is provable in principle. Consider the case where the fifth axiom of Euclid (about one unique parallel line) has been tried to be proven in Euclid's system[9] although it is independent. In this case, a valid proof couldn't be carried out. What happened was that one discovered alternative geometries, as Lobachevsky first did in Kazan 1826 (first published: [Lobachevsky 1829])[10]. Consider the case of the Axiom of Choice and the Generalized Continuum Hypotheses. After correct conjectures about their independence from the axioms of set theory (von Neumann) first the consistency with [Gödel 1940] and then the independency (i.e. the consistency of the negation, [Cohen 1963]) from these axioms has been proven.

Consider the proof of Fermat's Last Theorem: in this case a gap in the proof by Wiles has been discovered by Ribet. However Wiles filled the gap after one year (in 1994).

The upshot is: in difficult proofs of logic or mathematics *CQ* and CQ_1 have to be applied to every step of the proof, i.e. every step has to be critically controlled.

2.2.4. K_4: Verification

K_4 does not concern universal hypotheses, laws or theories. It is evident that one cannot test all the instances of a universal hypotheses or laws (less of a theory) and so one cannot verify them. Verification used for a "Sinnkriterium" (the sense of a statement is its method of verification) was an idea of the early Vienna Circle but was soon given up for its absurd consequence that universal hypotheses and laws would become meaningless (senseless).

Verification is concerned therefore only with statements representing single events by direct or indirect observation or by experiment. But even then verification is questionable since the requirement of science is (1) a reproducible effect which involves many single events, (2) an observer-invariant event and (3) a suitable interpretation of the observational data with the help scientific hypotheses, laws or theories.

Therefore Popper defended that there is no verification at all. We do not go that far but accept verification of basic statements about singular events in a restricted sense as follows:

Ad (1) Reproducible effect:

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9 Even Gauss attempted to do that until 1804, as a letter to the father of J. Bolyai of 1804 shows: cf. [Meschkowski 1978, p. 28ff.] and [Bonola 1955, p. 84ff].

10 There is absolute, elliptic, hyperbolic, non-Archimedean, non-Euclidean, Riemannian geometry. Cf. Mainzer 2004.

There are singular events which are not reproducible but can be tested nevertheless: a car accident, a star-explosion, a constellation of stars or planets which occur only seldom (though periodic) as for example a certain state of the perihelion of Mercury.

If the effect is reproducible, at least under similar conditions, then the experimental report does not consist of a statement representing a singular event but of a restricted universal statement representing a series of repeated experimental results, i.e. a scientific hypothesis describing an "effect". This point has been stressed by several philosophers of science, notably by Popper [Popper 1959, p. 45f., 86f.]. The main point is here that the description of the "effect" is not identical with the conjunction of several statements about singular events at a certain time under certain circumstances but abstracts from several individual conditions and averages the measurement-results. Therefore it is justified to speak of a restricted universal hypothesis; consequently the concept of *verification* has to be adapted accordingly.

Ad (2) Observer-Invariance:

A result of an observation or experiment is observer-invariant if every experienced expert (in the respective domain of research) gets to the same result. This holds usually well for a great number of scientific results. For example from observation of stars with telescopes to observations of cells or smaller parts of matter with microscopes[11]. However, there are domains where the observer-invariance is more restricted. For example: it is a statistical fact that X-ray pictures (of a certain type of illness) do not receive the same reading (interpretation for the diagnosis) by all the physicians (working in this domain). Applying *CQ* leads to absurd empirical consequences if the interpretation corresponds to real facts but leads to alternative readings if what is observed is not unambiguous.

Ad (3) Suitable interpretation:

It is well-known that raw data are insufficient for science; they have to be interpreted with the help of already corroborated hypotheses, laws or theories. Applying *CQ* and CQ_2, i.e. asking what would happen if we give up interpretation A leads to considering other possible interpretations B, C, D... For example, the same X-ray-pictures of the molecular structure of the DNA where available to Linus Pauling and to Watson and Crick. But Pauling interpreted them in a way inconsistent to some chemical established facts, whereas Watson and Crick found the correct interpretation [Watson 1968, sections 22, p. 27–28].

If observations and experiments have to be interpreted with the help of already corroborated hypotheses or laws on the one hand and hypotheses and laws need for their corroboration the positive outcome of tests by observations and experiments on the other, there seems to be a dangerous circularity for corroboration and verification in general. However, examples from science show that first a kind of circularity of that sort in fact exists and is unavoidable but secondly that even in this case observations and experimental tests on the one hand and corroboration of

[11] We assume that verification concerns observers on earth or close to it (space-station) only. Therefore it is not necessary to deal with non-observer invariance in moved systems (either inertial or accelerated, gravitational).

the hypothesis or law on the other can still be reliable. This can be shown by the experimental tests for corroborating the Special Theory of Relativity. The underlying methodological assumptions were these:

(i) Physical measurements instruments (rods and clocks) are real physical objects, not ideal entities.

(ii) Because of (i) they have to obey physical laws. But which ones? According to the *Copenhagen Interpretation* the quantum-mechanical phenomena have to be measured by a measurement instrument "outside" of the QM-system which obeys the laws of Classical Mechanics. Einstein refused this view both for his Theory of Relativity and for Quantum Mechanics. Therefore he required (iii).

(iii) The measurement instruments applied to test the Special Theory of Relativity (SR) have to obey the laws of SR.

It is plain that assumption (iii) leads to a kind of circularity: the measurement instruments which are used to test SR presuppose and obey the laws of SR. Does this mean that such a test or such reasons are not reliable? As the facts show this is not the case and moreover reveals that it is the only way to test the predictions of SR, i.e. the time-dilatation and the mass-increase. The first was tested with the help of very accurate atomic clocks in airplanes orbiting around the earth [Hafele/Keating 1972] the second in huge particle accelerators.

2.2.5. K_5: Knowledge as justified true belief

As is clear from the definition (Def. 5) K_5 is based on K_3 or K_4 or K_6 or K_7. We need not to incorporate K_1 and K_2 since these two types are different from K_3–K_7 in several respects: they are better characterized as forms of "understanding" than as forms of belief. In this point they are similar with K_3. But K_1 and K_2 need no separate justification since the critical question is CQ_1, not CQ_2 as with K_3–K_7. Applying CQ and CQ_2 to K_5 leads to checking the reliability of the experts concerning K_3, K_4, K_6, K_7 (cases (1) – (3) of Def. 5) or checking my own reliance on K_3, K_4, K_6, K_7.

Example: Aristotle defended that the orbits of the celestial bodies (also of the planets) are circles because circular movement is the most regular and perfect movement and from it the best measure-unit of time can be taken. For the Pythagoreans there was also an aesthetic component in spherical perfection: "The dogma of spherical perfection and its cosmologic consequences may be considered the kernel of early Pythagorean science." [Sarton 1966, p. 212].

Galileo might have had also aesthetic reasons like the Pythagoreans. But the other – more important reason for Galileo – might have been his understanding that the laws of motion are rotationally symmetric and therefore allow circles as its simplest solutions even if circles are not required by the laws of motion. But what if he would have thought that the orbits are exclusively determined by the rotationally symmetric laws? Then he would have been fully justified to believe in the orbits as circles. In fact *initial conditions* in addition to the laws determine the orbits. And these initial conditions may be asymmetric, may break the symmetry; i.e. determine the deviation from circles to produce the ellipses observationally discovered by Tycho Brahe and theoretically described and explained by Kepler and Newton. Thus, applying CQ and CQ_2 opens the way for the observations of Tycho Brahe and for the theoretical questions of Kepler and Newton.

2.2.6. K_6: Knowledge as possessing epistemic entropy and information

Applying CQ and CQ_2 to K_6 concerns two parts: epistemic entropy (cf. Def. 6.1) and epistemic information (cf. Def. 6.2). And here again, we have to split up the application to singular statements and to law-statements.

Concerning singular statements representing singular events, critical questions concerning epistemic entropy have to be answered by K_4 and investigating consequences. Concerning law statements representing laws, the question of a higher or lower epistemic entropy is very important for the domain of application of the law.

Example: Euler's formulation of Newton's second law (of motion) treats mass as a constant: $F = m\ (d^2x\ /\ dt^2)$. No real possible states, where mass is dependent on velocity (i.e. relativistic mass) can satisfy this law statement; its domain is Classical Mechanics. However, Newton's formulation $F = d{\cdot}m{\cdot}v\ /\ d{\cdot}t$ has a larger epistemic entropy beyond Classical Mechanics; i.e. it tolerates relativistic mass, because it says that force is the time-dependency of the momentum (that means that mass is involved).

Applying CQ and CQ_2 to epistemic information, we see that the answer is more complicated. In case of a singular statement representing a particular event CQ_2 leads to investigating the excluded possible states and consequently to considering alternatives. This is similar if we apply CQ_2 to law-statements representing laws: the consideration of the excluded possible states leads to possible alternatives.

Example: "All planets move in circles around the sun" excludes that a planet's velocity is sometimes greater (faster), sometimes smaller (slower) on the circular orbit. But this is exactly what happens when the orbits are ellipses and the sun is in one true focus. This is described by Kepler's second law: the straight line from the sun to the planet coats surfaces in equal times; i.e. the planet moves faster when it is closer to the sun.

2.2.7. K_7: Knowledge as justified corroboration

Applying CQ leads to different considerations w. r. t. either cases of $K_{7.1}$ or cases of $K_{7.2}$.

When applied to cases of $K_{7.1}$, we know from cosmological investigations that the fundamental constants of nature cannot be even slightly different than they are, otherwise there would not be carbon based life and we would not be here [cf. Barrow/Tipler 1986]. Thus in this case – and also in the case of conservation laws or the law of entropy, we are in a situation of applying CQ_1 instead of CQ_2.

On the other hand when CQ is applied to $K_{7.2}$ for example to the Hardy-Weinberg law or to some new scientific hypothesis, we have to apply CQ_2. This is even the case with hypotheses which have been corroborated for a relatively long time.

Examples: For decades, it was a well corroborated hypothesis in astronomy that no single star can have a mass greater than 200 sun-masses. Accordingly to CQ_2, the consequences from giving up this hypothesis were hardly acceptable. Nevertheless, greater stars, which cannot be interpreted as heaps of stars, have been observed recently. Another hypothesis, well corroborated for decades was this: "All elementary particles consist either of 2 quarks (mesons) or 3 quarks (baryons: protons and neutrons)." Also here CQ_2 was applied and it was considered what consequences would follow if there were particles with more quarks. In fact, some respective anomalies have been discovered already in 2008 by the Belle-group in Japan. The above hypothesis was refuted in 2014 when particles with 4 quarks were discovered [PRL 2014, 222002; cf. Lange, Uwer 2014].

Another example is the well-corroborated value of the gene-mutation rate of the human DNA which is caused by mistakes in the DNA-replication (not by environment like strong ultraviolet or X-rays). It is between $1{:}10^5$ to $1{:}10^8$.The hypothesis was that these are the only mistakes made in replication. This was refuted by the discovery that many more mistakes are permanently made but are immediately repaired by special enzymes (Nobel Prize in Chemistry of 2015 for Lindahl, Modrich and Sancar [cf. Fischer 2015]) such that the "mutation-rate" shows only the remaining mistakes which are extremely little. Moreover the Nobel Laureates found out that the mistakes cannot have been repaired by *chance*, otherwise at every human cell division more than 3000 new mutations would occur.

CQ and CQ_2 are also applicable to basic preconditions of some scientific domain. In such a way Einstein has questioned the following three preconditions of the Galilei invariance of Classical Mechanics.

The time scale is the same in all inertial systems.

Simultaneity is the same in all inertial systems.

The spatial distance of two simultaneous events is the same in all inertial systems.

Giving up these preconditions and consequently giving up absolute time and absolute space led to the Special Theory of Relativity [cf. Mittelstaedt/Weingartner 2005, p. 120ff.].

При каких условиях знание является критическим?

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В данной статье авторы защищают плюрализм относительно понятия знания и показывают, что семь типов знания, которые они различают, могут быть объединены критериями, которые характеризуют типы знания как критические. В первой части статьи дается классификация семи типов знания: 1. знание как истинное непосредственно объективное понимание (например, простые логические или математические истины); 2. знание как истинное непосредственное понимание (например, cogito ergo sum); 3. знание как истинное объективное понимание через доказательство (например, строгое доказательство в логике, математике и науке); 4. знание как верификация (например, прямое или непрямое наблюдение или эксперимент); 5. знание как истинное мнение с обоснованием (например, вера в экспертов); 6. знание как обладание эпистемической энтропией и информацией (например, возможные состояния, которые отвечают гипотезе h и возможные состояния, которые запрещаются гипотезой h); 7. знание

как обоснованное подтверждение (например, второй закон Кеплера подтверждается строгими тестами). Во второй части предлагаются условия для того, чтобы знание могло являться критическим. Основной критический вопрос (КВ) таков: что случится если p, утверждение, которое мы считаем знанием, будет отброшено? Этот вопрос затем прилагается ко всем семи типам знания. Первый и второй типы знания не могут быть отброшены, поскольку это приведет к противоречию и абсурду. Применение КВ к третьему типу знания может намекать на нахождение неправильного шага или разрыва в доказательстве. КВ в применении к четвертому типу знания ведет к критическому исследованию, является ли верификация воспроизводимой, является ли она независимой от наблюдателя, имеет ли она подходящую интерпретацию с помощью хорошо обоснованной гипотезы или закона. Применение КВ к пятому типу знания заставляет проверить надежность экспертов или свою собственную уверенность. Более того, это ведет к открытию скрытых предположений и предпосылок. Применение КВ к шестому типу знания ведет к исследованию исключенных возможных состояний и, как следствие, к рассмотрению альтернатив. Например, евклидова геометрия исключает положительную или отрицательную кривизну; альтернативная (гиперболическая) геометрия с отрицательной кривизной была открыта Лобачевским и Бойяи. Применяя КВ к седьмому типу знания, мы должны различить два случая. Хорошо подтвержденные законы природы или фундаментальные природные константы ведут себя схоже с первым типом знания: если мы их отбросим, то мы столкнемся с абсурдными последствиями. С другой стороны, применение КВ к обоснованным научным гипотезам часто ведет к их ревизии, и иногда к их опровержению.

Ключевые слова: типы знания, эпистемология, условия знания

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